Abstract

This thesis contains the study of generalizations of Armendariz rings. Recall that a ring $R$ is called an Armendariz ring if whenever $p(x) = \sum_{i=0}^{m} a_{i}x^{i}$ and $q(x) = \sum_{j=0}^{n} b_{j}x^{j} \in R[x]$ such that $p(x)q(x) = 0$ implies $a_{i}b_{j} = 0$ for each $0 \leq i \leq m$ and $0 \leq j \leq n$. A central goal of the work is to study the various Armendariz-like conditions passing from rings to certain kinds of over-rings, such as polynomial rings, Laurent polynomial rings, upper-triangular matrix rings, Dorroh extensions, corners, direct product of rings, direct sums and sub-direct sums. We define almost Armendariz ring as if whenever $p(x) = \sum_{i=0}^{m} a_{i}x^{i}$ and $q(x) = \sum_{j=0}^{n} b_{j}x^{j} \in R[x]$ such that $p(x)q(x) = 0$ implies $a_{i}b_{j} \in N_{*}(R)$ for each $0 \leq i \leq m$ and $0 \leq j \leq n$. Clearly, Armendariz ring is a special case of an almost Armendariz ring.

Here, it is shown that if $I$ is a semicommutative ideal of $R$ and $R/I$ is almost Armendariz, then $R$ is almost Armendariz. Also, we prove that a ring $R$ is almost Armendariz iff $R[x]$ is almost Armendariz, which is an analogue of Theorem 2 of Anderson and Camillo\textsuperscript{3}. Further, we extend it as power serieswise almost Armendariz ring and study its structural properties, and usefulness.

In Chapter 3, we introduce weak ideal-Armendariz ring which unifies the Armendariz ring and weakly semicommutative ring. Here, we generalize weak ideal-Armendariz as strongly nil-IFP and some examples and properties are discussed which distinguishes it from other existing structures. It is prove that if $I$ is a semicommutative ideal of $R$ and $R/I$ is strongly nil-IFP, then $R$ is strongly nil-IFP. Moreover, if $R$ is 2-primal, then $R[x]/<x^{n}>$ is strongly nil-IFP.

In continuation, based on the concept of the lower nil $M$-Armendariz ring introduced by Alhevaz et al.\textsuperscript{11}, for a strictly totally ordered monoid $M$, and a semicommutative ideal $I$, we
prove that if $R/I$ is lower nil $M$-Armendariz, then $R$ is lower nil $M$-Armendariz. Moreover, if $M$ is a monoid, $R$ is 2-primal $M$-Armendariz and $N$ a u.p. monoid, then $R[N]$ is lower nil $M$-Armendariz.

In Chapter 5, we extend our study of almost Armendariz by using the concept of skew polynomial ring and define $\alpha$-almost Armendariz and $\alpha$-skew almost Armendariz ring, respectively. Here, first we show that $\alpha$-compatible 2-primal ring is $\alpha$-almost Armendariz. Also, $R$ is $\alpha$-almost Armendariz iff $R[x]$ is $\alpha$-almost Armendariz, where $\alpha^k = I_R$ for some $k \in \mathbb{Z}_+$. In Section 5.2, we consider a reversible ring $R$ and show that for $a, b \in R$ and an endomorphism $\alpha$ on $R$, if $ab = 0$ implies $a\alpha(b) = 0$, then $R$ is $\alpha$-skew almost Armendariz. Again, in Section 5.3, consider a $\alpha$-derivation $\delta$ on the ring $R$, which increases difficulty level, and introduce almost $(\alpha, \delta)$-skew Armendariz. For a $\alpha$-rigid $\delta$-ideal $N_\alpha(R)$ of $R$, we prove that if $N_\alpha(R[x; \alpha, \delta])$ is completely semiprime ideal of $R[x; \alpha, \delta]$, then $R[x; \alpha, \delta]$ is almost Armendariz. Finally, we define the skew power-serieswise almost Armendariz ring and also study its natural extension which is a skew Laurent polynomial ring $R[x, x^{-1}; \alpha]$ and prove some analogues results like above sections. We mention here one specific result as follows:

Let $R$ be an $\alpha$-compatible such that $rx^ps = sx^p \forall r \in R, s \in S, p \geq 1$. Let $D$ be the Dorroh extension and $\alpha$, an $S$-endomorphism of $R$. If $R$ is skew power-serieswise almost Armendariz iff $D$ is skew power-serieswise almost Armendariz.